Series Questions

For each, state True or False. If the statement is false, give a counter example.

1. If $\lim_{n \to \infty} a_n = 0$, then $\sum a_n$ converges.

2. If b_n is decreasing to zero, then $\sum (-1)^n b_n$ converges.

3. If b_n is decreasing to zero, then $\sum (-1)^n b_n$ converges absolutely.

4. If $a_n < b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.

5. If $|a_n| < b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.

6. If $\lim_{n \to \infty} \left| \dfrac{a_{n+1}}{a_n} \right| = 1$, then the series diverges.

7. If a series converges, then it is absolutely convergent.

8. If a series is absolutely convergent, then it converges.

9. If $\displaystyle\sum_{n=2}^{\infty} a_n$ diverges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

10. If $\displaystyle\sum_{n=2}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

11. $\left(\displaystyle\sum_{n=1}^{\infty} a_n \right)^2 = \displaystyle\sum_{n=1}^{\infty} (a_n)^2$.